Power and Spectral Efficiency of Multi-Pair Massive Antenna Relaying Systems with Zero-Forcing Relay Beamforming

Tuyet Van Thi Le and Yun Hee Kim

***Abstract*—We consider a multi-pair multi-antenna relay system, delivering data from multi-sources to their destinations simultaneously with the help of a massive antenna relay. We analyze the spectral efficiency and power scaling laws under perfect and imperfect channel state information (CSI) for zero-forcing relay beamforming. The analysis shows that the transmit power can be scaled down by $1/M^{\alpha}$ at the users and by $1/M^{\beta}$ at the relay as the number M of relay antennas increases, where $(\alpha, \beta) = (t, 1)$ or $(1, t)$ for $0 \leq t \leq 1$ in the case of perfect CSI and $(\alpha, \beta) = (t, 1-t)$ or $(1/2, t)$ for $0 \leq t \leq 1/2$ in the case of imperfect CSI.**

***Index Terms*—Massive antennas, multiuser relay communications, power scaling law, spectral efficiency, zero forcing**

I. Introduction

Multi-antenna relays have been introduced to multi-device communication systems to coordinate the data transfer among the devices in an efficient way [1], [2]. Such high complexity relays can be deployed for machine-type communications to support the concurrent data transfer among the device pairs in local areas [3], which will be called multi-pair multi-antenna relay (MP-MAR) system in the sequel. The MP-MAR system is closely related to a multiuser multiple-input multiple-output (MU-MIMO) system since the relay in a MP-MAR system has a complexity comparable to the BS in MU-MIMO.

Recently, massive MU-MIMO employing large-scale antenna arrays at a BS has received huge interest due to the capability of interference suppression and power saving even with suboptimal linear receivers [4]–[6]. Maximal ratio transmission (MRT) and zero-forcing (ZF) beamforming have been studied for the downlink of a massive MU-MIMO system with perfect and imperfect channel state information (CSI) [5]. Similarly, maximal ratio combining (MRC), ZF, and minimum mean square error (MMSE) beamforming have been studied for the uplink, where the power scaling law is derived to quantify the power efficiency of massive BS antennas [6]. Like massive MU-MIMO, a relay with massive antenna arrays can be deployed in the MP-MAR system as in [7]–[9]. The studies in [7], [8] obtain respectively the sum rate of one-way relaying (OWR) and two-way relaying (TWR) with infinite number of relay antennas when MRC/MRT and ZF/ZF are employed for receive/transmit relay beamforming under perfect CSI. The achievable sum rate achievable with a finite number of relay antennas is derived in [9], but it is obtained only for TWR with MRC/MRT relay beamforming when perfect CSI is available at both the relay and destinations.

This letter revisits the massive MP-MAR system of [7] with ZF/ZF relay beamforming, which is attractable for finite numbers of relay antennas from a performance perspective. For both cases of perfect and imperfect CSI at the relay, we provide a more accurate and comprehensive analysis on the system performance in terms of the spectral efficiency (SE) valid for any finite number of relay antennas and the power scaling laws for large number of relay antennas.

Notations: The superscripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, and $(\cdot)^{-1}$ denote the transpose, complex conjugate, Hermitian transpose, and inverse, respectively. The matrix $\boldsymbol{I}_n$ represents the $n \times n$ identity matrix, $\mathbf{0}_{m\times n}$ indicates the $m \times n$ all-zero matrix, and $\operatorname{diag}(\boldsymbol{a})$ represents the diagonal matrix having $\boldsymbol{a}$ for the diagonal vector. The operations $\operatorname{Tr}\{\boldsymbol{A}\}$, $\|\boldsymbol{A}\|_F$, and $[\boldsymbol{A}]_{k,l}$ represent the trace, Frobenius-norm, and (k,l)th element of a matrix $\boldsymbol{A}$, respectively. In addition, $\sim$ denotes 'distributed as', $\mathbb{E}[\cdot]$ denotes the expectation, $\boldsymbol{R}_{\boldsymbol{ab}} = \mathbb{E}[\boldsymbol{a}\boldsymbol{b}^H]$ refers to the vector correlation matrix, and $\mathcal{CN}(\bar{\boldsymbol{A}}, \boldsymbol{\Sigma})$ indicates the complex Gaussian distributed matrix with mean $\bar{\boldsymbol{A}} = \mathbb{E}[\boldsymbol{A}]$ and covariance matrix $\boldsymbol{\Sigma} = \mathbb{E}\left[\operatorname{vec}\left(\boldsymbol{A} - \bar{\boldsymbol{A}}\right)\operatorname{vec}\left(\boldsymbol{A} - \bar{\boldsymbol{A}}\right)^H\right]$, where $\operatorname{vec}(\boldsymbol{A})$ denotes the column vector obtained by stacking the columns of a matrix $\boldsymbol{A}$.

II. System Model

A. Overall Description

Consider the massive MP-MAR system in Fig 1, which consists of K device pairs $\{(\mathrm{S}_k, \mathrm{D}_k)\}_{k=1}^K$ and a relay R. The infrastructure-deployed relay is equipped with M ($\gg K$) antennas whilst the sensor-like devices are equipped with a single antenna. The relay help source devices $\{\mathrm{S}_k\}_{k=1}^K$ deliver data to their destinations $\{\mathrm{D}_k\}_{k=1}^K$ in two time phases. The channel responses from the sources to the relay and from the relay to the destinations are arranged in matrices $\boldsymbol{G}_1 = [\boldsymbol{g}_{1,1}\ \boldsymbol{g}_{1,2}\ \cdots\ \boldsymbol{g}_{1,K}] \in \mathbb{C}^{M\times K}$ and $\boldsymbol{G}_2^T = [\boldsymbol{g}_{2,1}\ \boldsymbol{g}_{2,2}\ \cdots\ \boldsymbol{g}_{2,K}]^T \in \mathbb{C}^{K\times M}$, respectively, with $\boldsymbol{g}_{1,k}$ and $\boldsymbol{g}_{2,k}$ denoting the $M \times 1$ channel vectors between source k and the relay and between the relay and destination k, respectively. The channel matrices are modeled as

$$\boldsymbol{G}_i = \boldsymbol{H}_i \boldsymbol{\Omega}_i^{1/2} \sim \mathcal{CN}(\mathbf{0}_{M\times K}, \boldsymbol{\Omega}_i \otimes \boldsymbol{I}_M) \tag{1}$$

for $i = 1, 2$, where $\boldsymbol{H}_i \sim \mathcal{CN}(\mathbf{0}_{M\times K}, \boldsymbol{I}_{MK})$ represents Rayleigh fading, $\boldsymbol{\Omega}_i = \operatorname{diag}([\omega_{i,1}\ \omega_{i,2}\ \cdots\ \omega_{i,K}])$ reflects the path-poss component, and $\otimes$ denotes the Kronecker product.

Manuscript received August 07, 2014. The associate editor coordinating the review of this letter and approving it for publication was M. Matthaiou.

This work was supported by the National Research Foundation of Korea, with funding from the Ministry of Education under Grants 2012R1A1A2040091.

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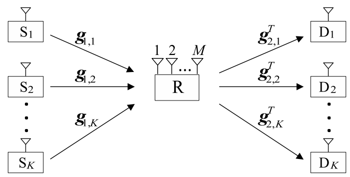

Fig. 1. Massive MP-MAR system.

In the first time phase, all sources transmit their symbols to the relay at transmit power $\mathcal{P}_{\mathrm{S}}$. The received signal at the relay is written as

$$\boldsymbol{z}_{\mathrm{R}} = \sqrt{\mathcal{P}_{\mathrm{S}}}\boldsymbol{G}_1\boldsymbol{x} + \boldsymbol{n}_{\mathrm{R}}, \tag{2}$$

where $\boldsymbol{x} = [x_1 \ x_2 \ \cdots \ x_K]^T \sim \mathcal{CN}(\mathbf{0}_{K\times 1}, \boldsymbol{I}_K)$ is the symbol vector from K sources and $\boldsymbol{n}_{\mathrm{R}} \sim \mathcal{CN}(\mathbf{0}_{M\times 1}, \mathcal{N}_0\boldsymbol{I}_M)$ is the noise vector at the relay. The relay transforms the received signal as

$$\boldsymbol{s}_{\mathrm{R}} = \boldsymbol{W}\boldsymbol{z}_{\mathrm{R}}, \tag{3}$$

where $\boldsymbol{W} = \sqrt{\frac{\mathcal{P}_R}{\theta}}\tilde{\boldsymbol{W}}$ is an $M \times M$ relay processing matrix with power scaling factor $\theta = \mathbb{E}\left[\left\|\tilde{\boldsymbol{W}}\boldsymbol{z}_{\mathrm{R}}\right\|^2\right]$ satisfying the relay power constraint $\mathcal{P}_{\mathrm{R}}$ in a long-term average.

In the second time phase, the relay forwards $\boldsymbol{s}_{\mathrm{R}}$ to the destinations. The received signals at the destinations can be arranged in a vector as

$$\boldsymbol{y}_{\mathrm{D}} = \sqrt{\mathcal{P}_{\mathrm{S}}}\boldsymbol{G}_2^T\boldsymbol{W}\boldsymbol{G}_1\boldsymbol{x} + \boldsymbol{G}_2^T\boldsymbol{W}\boldsymbol{n}_{\mathrm{R}} + \boldsymbol{n}_{\mathrm{D}}, \tag{4}$$

where $\boldsymbol{n}_{\mathrm{D}} \sim \mathcal{CN}(\mathbf{0}_{K\times 1}, \mathcal{N}_0\boldsymbol{I}_K)$ is the vector of the noises at the destinations.

B. Relay Processing with Imperfect CSI

The relay obtains CSI by performing linear MMSE (LMMSE) channel estimation using orthogonal pilot sequences of length τ_p transmitted at power $\mathcal{P}_{\mathrm{p}}$ [6]. The LMMSE channel estimate $\hat{\boldsymbol{G}}_i$ on $\boldsymbol{G}_i$ can be expressed as $\hat{\boldsymbol{G}}_i = \boldsymbol{G}_i + \boldsymbol{E}_i$, where $\boldsymbol{E}_i$ is the error matrix with $\mathbb{E}\left[\mathrm{vec}(\boldsymbol{E}_i)\mathrm{vec}(\hat{\boldsymbol{G}}_i)^H\right] = \mathbf{0}_{MK\times MK}$. Since $\boldsymbol{G}_i \sim \mathcal{CN}(\mathbf{0}_{M\times K}, \boldsymbol{\Omega}_i \otimes \boldsymbol{I}_M)$, $\hat{\boldsymbol{G}}_i$ and $\boldsymbol{E}_i$ are also zero-mean complex Gaussian with vector correlation

$$\boldsymbol{R}_{\mathrm{vec}(\hat{\boldsymbol{G}}_i)\mathrm{vec}(\hat{\boldsymbol{G}}_i)} = \boldsymbol{R}_{\mathrm{vec}(\boldsymbol{G}_i)\mathrm{vec}(\hat{\boldsymbol{G}}_i)} = \hat{\boldsymbol{\Omega}}_i \otimes \boldsymbol{I}_M \tag{5}$$

and $\boldsymbol{R}_{\mathrm{vec}(\boldsymbol{E}_i)\mathrm{vec}(\boldsymbol{E}_i)} = \boldsymbol{\Xi}_i \otimes \boldsymbol{I}_M$, respectively. Here, the correlation matrices are determined by the diagonal matrices $\hat{\boldsymbol{\Omega}}_i = \boldsymbol{\Omega}_i\left(\boldsymbol{\Omega}_i + \frac{\mathcal{N}_o}{\tau_{\mathrm{p}}\mathcal{P}_{\mathrm{p}}}\boldsymbol{I}_K\right)^{-1}\boldsymbol{\Omega}_i$ with $\hat{\omega}_{i,k} = \frac{\omega_{i,k}^2}{\omega_{i,k} + \mathcal{N}_0/(\tau_{\mathrm{p}}\mathcal{P}_{\mathrm{p}})}$ for the kth diagonal element and $\boldsymbol{\Xi}_i = \left(\boldsymbol{\Omega}_i^{-1} + \frac{\tau_{\mathrm{p}}\mathcal{P}_{\mathrm{p}}}{\mathcal{N}_o}\boldsymbol{I}_K\right)^{-1}$ with $\sigma_{i,k}^2 = \frac{\omega_{i,k}}{\omega_{i,k}\tau_{\mathrm{p}}\mathcal{P}_{\mathrm{p}}/\mathcal{N}_0 + 1}$ for the kth diagonal element.

With imperfect CSI $\{\hat{\boldsymbol{G}}_i\}_{i=1}^2$, the relay processing matrix based on the ZF criterion for receiving and transmitting is given by

$$\hat{\boldsymbol{W}}_{\mathrm{zf}} = \sqrt{\frac{\mathcal{P}_{\mathrm{R}}}{\theta_{\mathrm{zf}}}}\hat{\boldsymbol{G}}_2^*\left(\hat{\boldsymbol{G}}_2^T\hat{\boldsymbol{G}}_2^*\right)^{-1}\left(\hat{\boldsymbol{G}}_1^H\hat{\boldsymbol{G}}_1\right)^{-1}\hat{\boldsymbol{G}}_1^H, \tag{6}$$

where

$$\begin{aligned}\theta_{\mathrm{zf}} &= \mathbb{E}\left[\left\|\hat{\boldsymbol{G}}_2^*\left(\hat{\boldsymbol{G}}_2^T\hat{\boldsymbol{G}}_2^*\right)^{-1}\left(\hat{\boldsymbol{G}}_1^H\hat{\boldsymbol{G}}_1\right)^{-1}\hat{\boldsymbol{G}}_1^H\boldsymbol{z}_{\mathrm{R}}\right\|^2\right]\\ &= \mathcal{P}_{\mathrm{S}}\mathrm{Tr}\left\{\boldsymbol{\Psi}_2^T\right\} + \left(\mathcal{P}_{\mathrm{S}}\mathrm{Tr}\{\boldsymbol{\Xi}_1\} + \mathcal{N}_0\right)\mathrm{Tr}\left\{\boldsymbol{\Psi}_1\boldsymbol{\Psi}_2^T\right\},\end{aligned} \tag{7}$$

with $\boldsymbol{\Psi}_i = \mathbb{E}\left[\left(\hat{\boldsymbol{G}}_i\hat{\boldsymbol{G}}_i^H\right)^{-1}\right]$ for $i = 1, 2$.

C. End-to-End Performance

Let us now obtain the signal to interference-and-noise ratio (SINR) at the destinations with imperfect CSI. The received signal (4), after replacing $\boldsymbol{G}_i$ by $\hat{\boldsymbol{G}}_i - \boldsymbol{E}_i$ and $\boldsymbol{W}$ by $\hat{\boldsymbol{W}}_{\mathrm{ZF}}$, can be arranged as

$$\boldsymbol{y}_{\mathrm{D}} = \boldsymbol{s} + \boldsymbol{f} + \tilde{\boldsymbol{n}}, \tag{8}$$

where

$$\boldsymbol{s} \triangleq \sqrt{\mathcal{P}_{\mathrm{S}}}\hat{\boldsymbol{G}}_2^T\hat{\boldsymbol{W}}_{\mathrm{zf}}\hat{\boldsymbol{G}}_1\boldsymbol{x} = \sqrt{\mathcal{P}_{\mathrm{S}}\mathcal{P}_{\mathrm{R}}/\theta_{\mathrm{zf}}}\boldsymbol{x}, \tag{9}$$

$$\boldsymbol{f} \triangleq \sqrt{\mathcal{P}_{\mathrm{S}}}\left(\boldsymbol{E}_2^T\hat{\boldsymbol{W}}_{\mathrm{zf}}\boldsymbol{E}_1 - \hat{\boldsymbol{G}}_2^T\hat{\boldsymbol{W}}_{\mathrm{ZF}}\boldsymbol{E}_1 - \boldsymbol{E}_2^T\hat{\boldsymbol{W}}_{\mathrm{ZF}}\hat{\boldsymbol{G}}_1\right)\boldsymbol{x}, \tag{10}$$

and

$$\tilde{\boldsymbol{n}} \triangleq \hat{\boldsymbol{G}}_2^T\hat{\boldsymbol{W}}_{\mathrm{ZF}}\boldsymbol{n}_{\mathrm{R}} - \boldsymbol{E}_2^T\hat{\boldsymbol{W}}_{\mathrm{ZF}}\boldsymbol{n}_{\mathrm{R}} + \boldsymbol{n}_{\mathrm{D}} \tag{11}$$

represent the desired signals, interference incurred by channel estimation errors, and effective noise at the destinations, respectively.

The interference and noise have zero mean with correlation matrices

$$\begin{aligned}\mathbf{R}_{\boldsymbol{ff}} = \theta_{\mathrm{zf}}^{-1}\mathcal{P}_{\mathrm{S}}\mathcal{P}_{\mathrm{R}}\Big[&\mathrm{Tr}\{\boldsymbol{\Xi}_1\}\boldsymbol{\Psi}_1 + \mathrm{Tr}\{\boldsymbol{\Psi}_2^T\}\boldsymbol{\Xi}_2\\ &+ \mathrm{Tr}\{\boldsymbol{\Xi}_1\}\mathrm{Tr}\left\{\boldsymbol{\Psi}_1\boldsymbol{\Psi}_2^T\right\}\boldsymbol{\Xi}_2\Big]\end{aligned} \tag{12}$$

and

$$\mathbf{R}_{\tilde{\boldsymbol{n}}\tilde{\boldsymbol{n}}} = \theta_{\mathrm{zf}}^{-1}\mathcal{P}_{\mathrm{R}}\mathcal{N}_0\left[\boldsymbol{\Psi}_1 + \mathrm{Tr}\left\{\boldsymbol{\Psi}_1\boldsymbol{\Psi}_2^T\right\}\boldsymbol{\Xi}_2\right] + \mathcal{N}_0\boldsymbol{I}_K. \tag{13}$$

The effective SINR at destination D_k is then given by

$$\begin{aligned}\Gamma_k &= \frac{\mathcal{P}_{\mathrm{S}}\mathcal{P}_{\mathrm{R}}}{\theta_{\mathrm{zf}}\left[\boldsymbol{R}_{\boldsymbol{ff}}\right]_{k,k} + \theta_{\mathrm{zf}}\left[\boldsymbol{R}_{\tilde{\boldsymbol{n}}\tilde{\boldsymbol{n}}}\right]_{k,k}}\\ &= \frac{\mathcal{P}_{\mathrm{S}}\mathcal{P}_{\mathrm{R}}}{\zeta_{\mathrm{S}}\mathcal{P}_{\mathrm{R}}\left[\boldsymbol{\Psi}_1\right]_{k,k} + \mathcal{P}_{\mathrm{S}}\zeta_{\mathrm{R},k}\mathrm{Tr}\left\{\boldsymbol{\Psi}_2^T\right\} + \zeta_{\mathrm{S}}\zeta_{\mathrm{R},k}\mathrm{Tr}\left\{\boldsymbol{\Psi}_1\boldsymbol{\Psi}_2^T\right\}},\end{aligned} \tag{14}$$

where $\zeta_{\mathrm{S}} = \mathcal{P}_{\mathrm{S}}\mathrm{Tr}\{\boldsymbol{\Xi}_1\} + \mathcal{N}_0$ and $\zeta_{\mathrm{R},k} = \mathcal{P}_{\mathrm{R}}\sigma_{2,k}^2 + \mathcal{N}_0$.

III. Spectral Efficiency

This section analyzes the sum rate and spectral efficiency (SE) of the massive MP-MAR system by applying the approach in [10] which assumes that interference and noise are independent complex Gaussian random variables. The achievable sum rate of the system is then lower-bounded as

$$R \geq R_L = \frac{1}{2}\sum_{k=1}^{K}\log_2(1 + \Gamma_k) \tag{15}$$

from which a bound on the SE incorporating the pilot overhead is also derived as

$$S_{\text{eff}} = \frac{T-\tau_{\text{p}}}{T} R \geq \frac{T-\tau_{\text{p}}}{2T} \sum_{k=1}^{K} \log_2(1+\Gamma_k) \qquad (16)$$

with pilot period T determined by the coherence time of the channel.

Let us now represent the effective SINR (14) explicitly by deriving $\boldsymbol{\Psi}_i = \mathbb{E}\left\{\left(\hat{\boldsymbol{G}}_i \hat{\boldsymbol{G}}_i^H\right)^{-1}\right\}$. Recall that the columns of $K \times M$ matrix $\hat{\boldsymbol{G}}_i^H$ are independent of each other and zero-mean complex Gaussian with covariance matrix $\hat{\boldsymbol{\Omega}}_i$. Hence, $\hat{\boldsymbol{G}}_i^H \hat{\boldsymbol{G}}_i$ is a $K \times K$ complex Wishart matrix with M degrees of freedom and parameter matrix $\hat{\boldsymbol{\Omega}}_i$ and $\left(\hat{\boldsymbol{G}}_i^H \hat{\boldsymbol{G}}_i\right)^{-1}$ becomes a $K \times K$ complex inverse Wishart matrix with M degrees of freedom and parameter matrix $\hat{\boldsymbol{\Omega}}_i^{-1}$ [11]. Therefore, we have

$$\boldsymbol{\Psi}_i = \frac{1}{M-K} \hat{\boldsymbol{\Omega}}_i^{-1}, \ i = 1, 2 \qquad (17)$$

which leads to

$$\Gamma_k = \frac{\mathcal{P}_{\text{S}} \mathcal{P}_{\text{R}} (M-K)^2}{(M-K)\left(\frac{\zeta_{\text{S}} \mathcal{P}_{\text{R}}}{\hat{\omega}_{1,k}} + \sum_{l=1}^{K} \frac{\mathcal{P}_{\text{S}} \zeta_{\text{R},k}}{\hat{\omega}_{2,l}}\right) + \sum_{l=1}^{K} \frac{\zeta_{\text{S}} \zeta_{\text{R},k}}{\hat{\omega}_{1,l} \hat{\omega}_{2,l}}}. \qquad (18)$$

In a special case of perfect CSI with $\hat{\omega}_{i,l} = \omega_{i,l}$ and $\sigma_{i,l}^2 = 0$, (18) becomes

$$\Gamma_k^{\text{perf}} = \frac{\mathcal{P}_{\text{S}} \mathcal{P}_{\text{R}} (M-K)^2}{(M-K)\left(\frac{\mathcal{P}_{\text{R}} \mathcal{N}_0}{\omega_{1,k}} + \sum_{l=1}^{K} \frac{\mathcal{P}_{\text{S}} \mathcal{N}_0}{\omega_{2,l}}\right) + \sum_{l=1}^{K} \frac{\mathcal{N}_0^2}{\omega_{1,l} \omega_{2,l}}}. \qquad (19)$$

Thus, the SE with perfect CSI is also obtained by plugging $\tau_{\text{p}} = 0$ and $\Gamma_k = \Gamma_k^{\text{perf}}$ in (16).

IV. Power Scaling Laws

We now quantify how much power can be scaled down at the devices and the relay as M grows large for a non-vanishing SE. For this purpose, we rewrite the powers as $\mathcal{P}_{\text{S}} = \mathcal{P}_{\text{p}} = \frac{E_{\text{S}}}{M^\alpha}$ and $\mathcal{P}_{\text{R}} = \frac{E_{\text{R}}}{M^\beta}$, where E_{S} and E_{R} are fixed constants and α and β are nonnegative real numbers indicting the power scaling laws at the devices and relay, respectively.

When $M \to \infty$, the effective SINR (18) can be approximated, from $M - K \approx M$, $\zeta_{\text{S}} \approx \zeta_{\text{R},k} \approx \mathcal{N}_0$, and $\hat{\omega}_{i,k} \approx \frac{\tau_{\text{p}} E_{\text{S}} \omega_{i,k}^2}{\mathcal{N}_0 M^\alpha}$, as

$$\Gamma_k \approx \frac{\frac{E_{\text{S}} E_{\text{R}}}{\mathcal{N}_0^2} M^{2-\alpha-\beta}}{a_1 M^{1+\alpha-\beta} + a_2 M + a_3 M^{2\alpha}}, \qquad (20)$$

where $a_1 = \frac{E_{\text{R}}}{\tau_{\text{p}} E_{\text{S}} \omega_{1,k}^2}$, $a_2 = \frac{1}{\tau_{\text{p}}} \sum_{l=1}^{K} \frac{1}{\omega_{2,l}^2}$, and $a_3 = \frac{\mathcal{N}_0^2}{\tau_{\text{p}}^2 E_{\text{S}}^2} \sum_{l=1}^{K} \frac{1}{\omega_{1,l}^2 \omega_{2,l}^2}$. For non-vanishing Γ_k with $M \to \infty$, we should have $\min(1-2\alpha, 1-\alpha-\beta, 2-3\alpha-\beta) \geq 0$ in (20), or equivalently

$$0 \leq \alpha \leq 1/2, \ \beta \geq 0, \ \alpha + \beta \leq 1. \qquad (21)$$

In particular, when $(\alpha, \beta) = (1/2, t)$ or $(t, 1-t)$ for $0 \leq t \leq 1/2$, Γ_k converges to a constant as $M \to \infty$: Specifically,

$$\Gamma_k \to \frac{\frac{E_{\text{S}} E_{\text{R}}}{\mathcal{N}_0^2}}{\frac{E_{\text{R}}}{\tau_{\text{p}} E_{\text{S}} \omega_{1,k}^2} + \frac{1}{\tau_{\text{p}}} \sum_{l=1}^{K} \frac{1}{\omega_{2,l}^2} + \frac{\mathcal{N}_0^2}{\tau_{\text{p}}^2 E_{\text{S}}^2} \sum_{l=1}^{K} \frac{1}{\omega_{1,l}^2 \omega_{2,l}^2}} \qquad (22)$$

when $(\alpha, \beta) = (1/2, 1/2)$, $\Gamma_k \to \frac{\tau_{\text{p}} E_{\text{S}}^2}{\mathcal{N}_0^2} \omega_{1,k}^2$ when $(\alpha, \beta) = (1/2, t)$ for $0 \leq t < 1/2$, and $\Gamma_k \to \frac{\tau_p E_{\text{S}} E_{\text{R}}}{\mathcal{N}_0^2} \left(\sum_{l=1}^{K} \frac{1}{\omega_{2,l}^2}\right)^{-1}$ when $(\alpha, \beta) = (t, 1-t)$ for $0 \leq t < 1/2$.

On the other hand, with perfect CSI, (19) becomes

$$\Gamma_k^{\text{perf}} \approx \frac{\frac{E_{\text{S}} E_{\text{R}}}{\mathcal{N}_0^2} M^{2-\alpha-\beta}}{\frac{E_{\text{R}}}{\mathcal{N}_0 \omega_{1,k}} M^{1-\beta} + \frac{E_{\text{S}}}{\mathcal{N}_0} \sum_{l=1}^{K} \frac{1}{\omega_{2,l}} M^{1-\alpha} + \sum_{l=1}^{K} \frac{1}{\omega_{1,l} \omega_{2,l}}} \qquad (23)$$

when $M \to \infty$, which requires $\min(1-\alpha, 1-\beta, 2-\alpha-\beta) \geq 0$, or equivalently

$$0 \leq \alpha \leq 1, \ 0 \leq \beta \leq 1, \ \alpha + \beta \leq 2 \qquad (24)$$

for non-vanishing Γ_k^{perf} with $M \to \infty$. In particular, Γ_k^{perf} converges to a constant as $M \to \infty$ when $(\alpha, \beta) = (1, t)$ or $(t, 1)$ for $0 \leq t \leq 1$: Specifically, we have

$$\Gamma_k^{\text{perf}} \to \frac{\frac{E_{\text{S}} E_{\text{R}}}{\mathcal{N}_0^2}}{\frac{E_{\text{R}}}{\mathcal{N}_0 \omega_{1,k}} + \frac{E_{\text{S}}}{\mathcal{N}_0} \sum_{l=1}^{K} \frac{1}{\omega_{2,l}} + \sum_{l=1}^{K} \frac{1}{\omega_{1,l} \omega_{2,l}}} \qquad (25)$$

when $(\alpha, \beta) = (1, 1)$, $\Gamma_k^{\text{perf}} \to \frac{E_{\text{S}}}{\mathcal{N}_0} \omega_{1,k}$ when $(\alpha, \beta) = (1, t)$ for $0 \leq t < 1$, and $\Gamma_k^{\text{perf}} \to \frac{E_{\text{R}}}{\mathcal{N}_0} \left(\sum_{l=1}^{K} \frac{1}{\omega_{2,l}}\right)^{-1}$ when $(\alpha, \beta) = (t, 1)$ for $0 \leq t < 1$.

The results imply that the power at the devices and relay can be scaled down simultaneously up to by $1/\sqrt{M}$ with imperfect CSI and by $1/M$ with perfect CSI. In the case of perfect CSI, Γ_k^{perf} is bounded for increasing M if either the power at the devices or the power at the relay is scaled down maximally ($\alpha = 1$ or $\beta = 1$). In case of imperfect CSI, Γ_k is bounded for increasing M if the power at the device is scaled down maximally ($\alpha = 1/2$): If the power at the device is not scaled down maximally ($\alpha < 1/2$), the power at the relay can be scaled further by $\beta = 1 - \alpha$ ($> 1/2$).

V. Numerical Results

The performance of the massive MP-MAR system is investigated through simulation and analysis with $\tau_{\text{p}} = 2K$ and $T = 400$ in symbols. All devices are assumed to be equidistant from the relay such that $\boldsymbol{\Omega}_i = \boldsymbol{I}_K$ for $i = 1, 2$. In the figure, SNR$= \mathcal{P}_{\text{S}}/\mathcal{N}_0$ is used as the reference parameter.

Fig. 2 shows the SE as a function of SNR when $K = 10$ and $\mathcal{P}_{\text{R}} = K\mathcal{P}_{\text{S}}$. In the figure, we compare the lower bounds on the SE (denoted by 'Anal.') given by (16) of the ZF/ZF with perfect and imperfect CSI with their simulation results (denoted by 'Simul.'). The figure shows that the lower bounds on the SE are almost indistinguishable from the simulation results even with finite M. For the benchmark, we also provide the performance of the MRC/MRT relay beamforming,

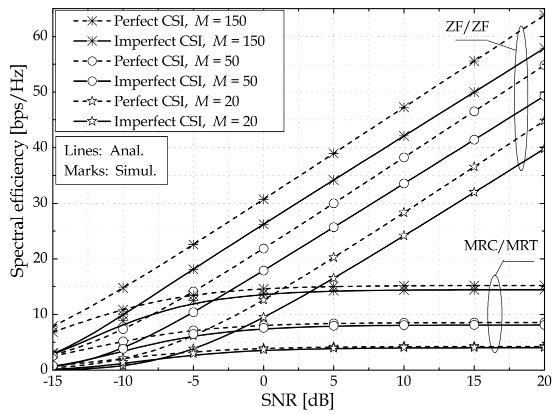

Fig. 2. SE as a function of SNR when $K = 10$, $\mathcal{P}_{\rm R} = K\mathcal{P}_{\rm S}$, and $\boldsymbol{\Omega}_i = \boldsymbol{I}_K$.

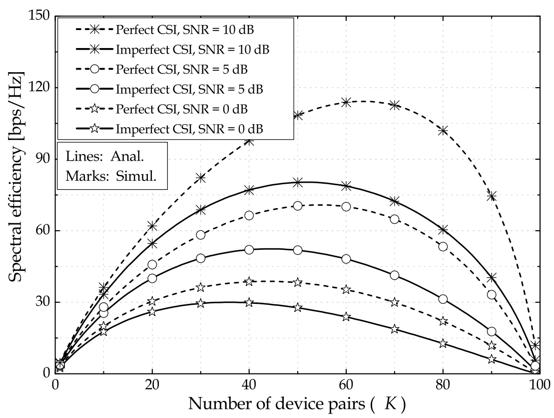

Fig. 3. SE as a function of the number K of device pairs when $M = 100$, $\mathcal{P}_{\rm R} = 2\mathcal{P}_{\rm S}$, and SNR = 0, 5, and 10 dB.

$\hat{\boldsymbol{W}}_{\rm mrx} = \sqrt{\frac{\mathcal{P}_{\rm R}}{\theta_{\rm mrx}}}\hat{\boldsymbol{G}}_2^*\hat{\boldsymbol{G}}_1^H$ with $\theta_{\rm mrx} = \mathbb{E}\left[\left\|\hat{\boldsymbol{G}}_2^*\hat{\boldsymbol{G}}_1^H\boldsymbol{z}_{\rm R}\right\|^2\right]$. Clearly, the SE increases with the number of relay antennas and SNR. In addition, the ZF/ZF outperforms the MRC/MRT significantly except for very low SNR values.

The SE is also shown for the number K of device pairs in Fig. 3 when $M = 100$ and $\mathcal{P}_{\rm R} = 2\mathcal{P}_{\rm S}$. The SE increases with K for small K since the gain from multiplexing the device signals is larger than the loss from the interference. The SE decreases with K for large K due to the opposite reason. Hence, an optimum number K^* maximizing the SE is observed, which tends to be smaller with imperfect CSI than with perfect CSI since the imperfect CSI incurs a larger interference. Clearly, the optimal K^* increases as the SNR increases since a larger SNR increases the gain from multiplexing the device signals.

The effect of power scaling (α, β) on the SE is shown in Fig. 4 when $K = 10$, $E_{\rm S}/\mathcal{N}_0 = 0$ dB, and $E_{\rm R}/\mathcal{N}_0 = 5$ dB. The SE with perfect CSI converges to a nonzero constant value as M increases when $(\alpha, \beta) = (1, 1)$, $(0, 1)$, and $(1, 0)$ while it grows without a bound when $(\alpha, \beta) = (1/3, 2/3)$ and $(1/2, 1/2)$. The SE with imperfect CSI diminishes to zero when $(\alpha, \beta) = (1, 1)$ and $(1, 0)$ as M increases while it converges to a nonzero constant value when $(\alpha, \beta) = (0, 1)$, $(1/3, 2/3)$, and $(1/2, 1/2)$ although the convergence rate is rather slow when $(\alpha, \beta) = (1/3, 2/3)$ and $(1/2, 1/2)$. We

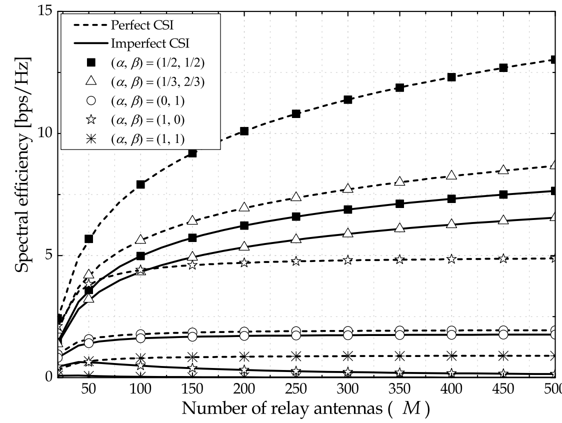

Fig. 4. SE as a function of the number M of relay antennas when $K = 10$, $E_{\rm S}/\mathcal{N}_0 = 0$ dB, and $E_{\rm R}/\mathcal{N}_0 = 5$ dB.

would like to note that the results of the figure comply with the power scaling laws derived in Section IV.

VI. Conclusions

We have analyzed the performance of a multi-pair massive antenna relaying system employing ZF/ZF relay beamforming. For the system, a tight bound on the spectral efficiency and the power scaling law are derived with perfect and imperfect CSI. The results show that the bounds agree with simulation results for a wide range of the number M of relay antennas. It is also observed that the power at both the relay and devices can be scaled down simultaneously up to by $\frac{1}{M}$ with perfect CSI and by $\frac{1}{\sqrt{M}}$ with imperfect CSI.

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